Chasing the Unicorn: the QGP & RHIC

Unicorn = fantastic and mythical beast!

Quark Gluon Plasma: deconfined, chirally symmetric matter

Q: Have AA collisions @ RHIC made the QGP?

Triumph of experiment: wealth of precise data

In central AA, some quantities change by ~ 5 from lower energies

Geometrical evidence: matter at high energy density "eats" jets

Exp. surprise: the (high-pt) tail wags the (low-pt) body of the Unicorn

Even qualitatively, *no* theory explains *all* interesting features.

A: *Some* type of QGP has been created

Hunting for the "Unicorn" @ SPS, RHIC, LHC, GSI

⇑Hunters = experimentalists. So: "All theorists are dogs..."

RHIC: Exp.'y, charm quarks "flow" (v2) like pions! sQGP?
"Most Perfect Fluid on Earth": Gyulassy, Heinz, Hirano, Teaney, Shuryak...

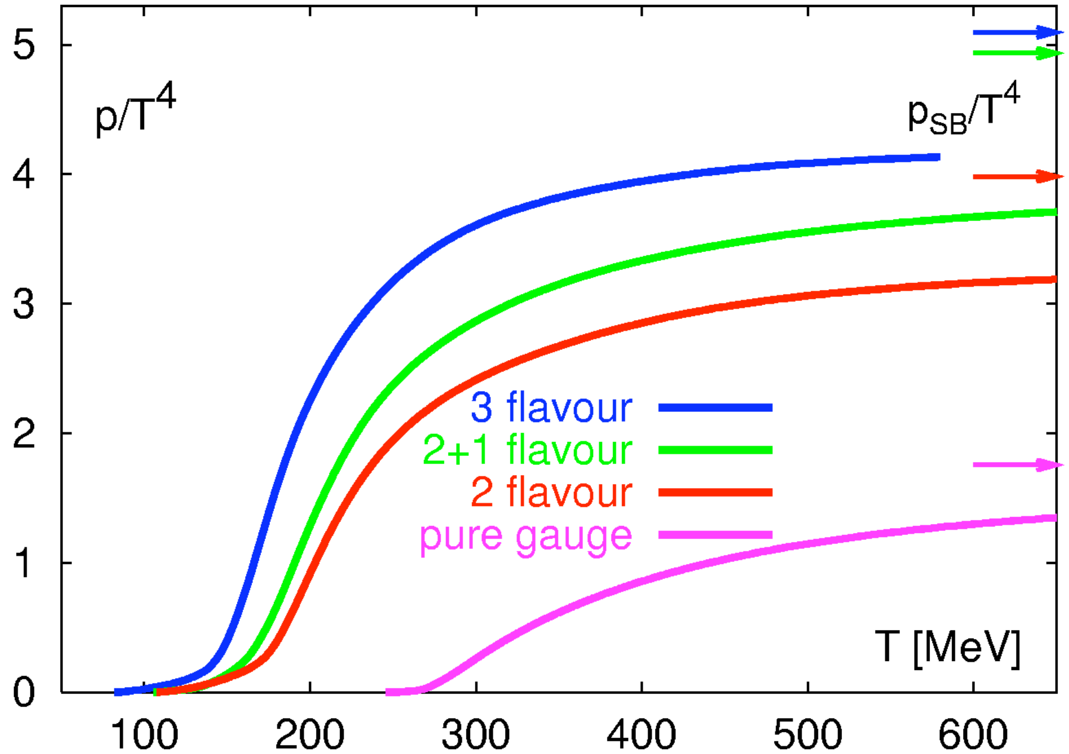
N.B.: but with *wrong* (bag) Equation of State!
Huovinen: v2 OK for bag EoS, but lattice EoS is as bad as purely hadronic EoS.

Lattice: SU(3) thermo., c & c/o quarks

With NO quarks: 1st order deconfining trans at T_d ≈ 270 MeV ± 5%

3 flavors of quarks: crossover, chiral sym restoration & deconfinement
T_chiral ~ 175 MeV

$\frac{p}{T^4} \uparrow$



Lattice:Bielefeld

$T \rightarrow$

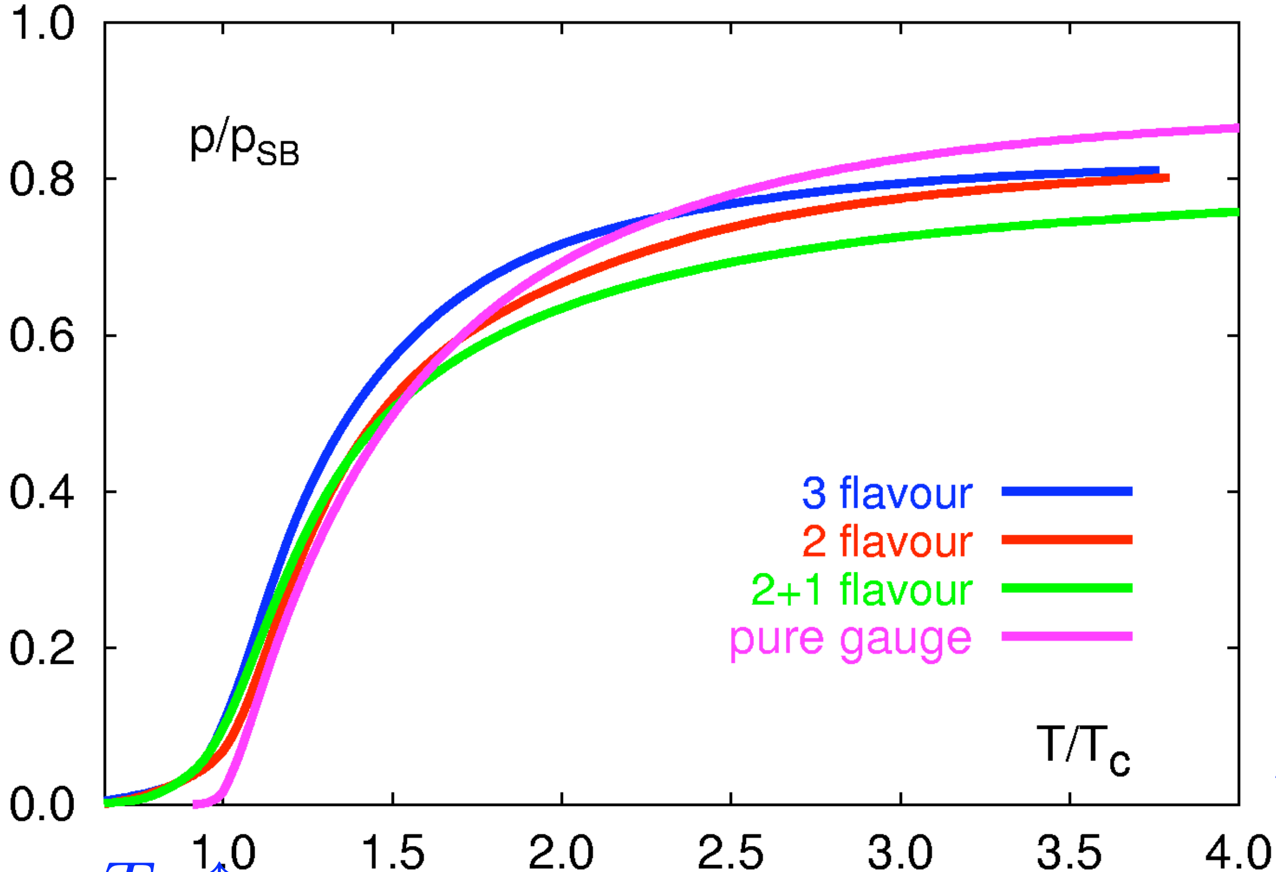
T = temperature, p(T) = pressure

Lattice SU(3) thermo.:“Flavor Independence”

Bielefeld: results are simple,
approximate universality:

$$\frac{p}{p_{ideal}}\left(\frac{T}{T_d}\right) \approx \text{universal}$$

$\frac{p}{p_{ideal}} \Uparrow$



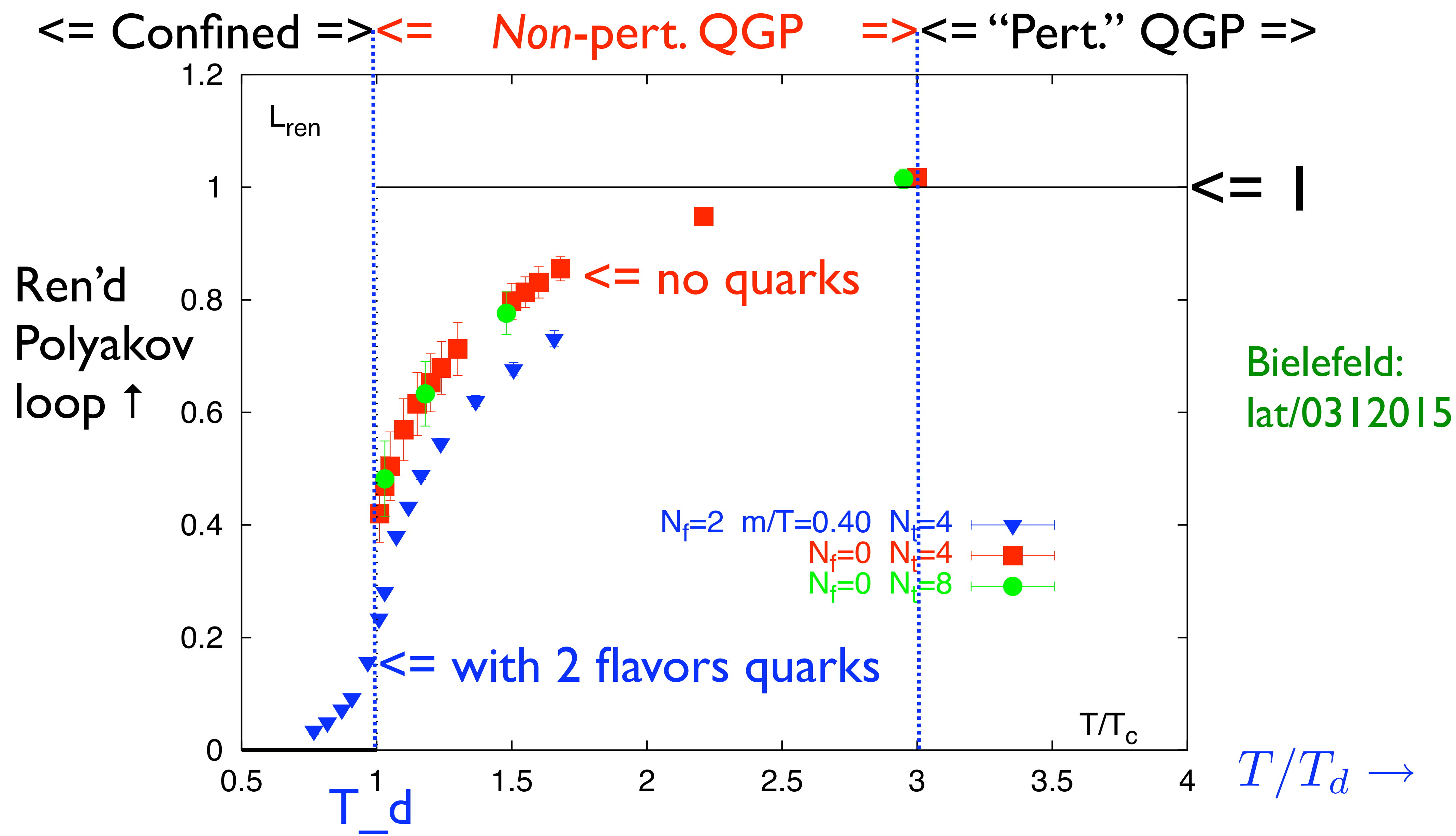
$T/T_d \rightarrow$

$T_d \uparrow$

Perhaps: even with quarks,“transition” *dominated by gluons*
=> Polyakov loops (matrix model)

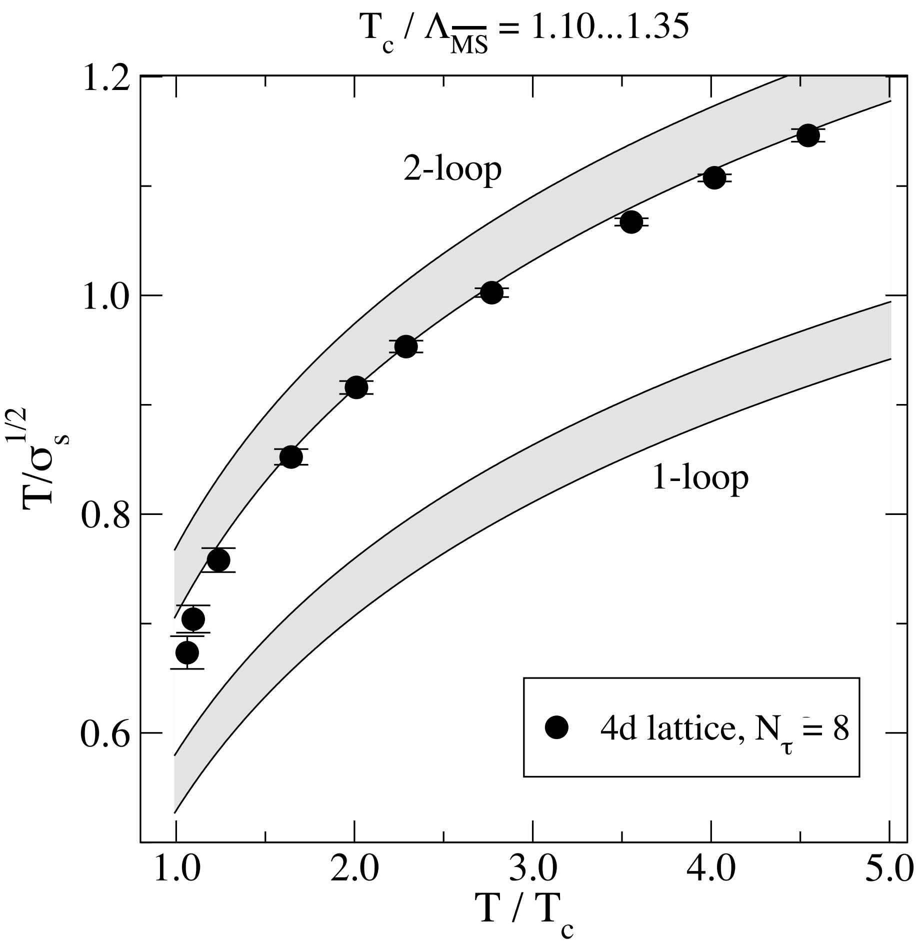
Non-perturbative QGP, Td ⇒ 3 Td

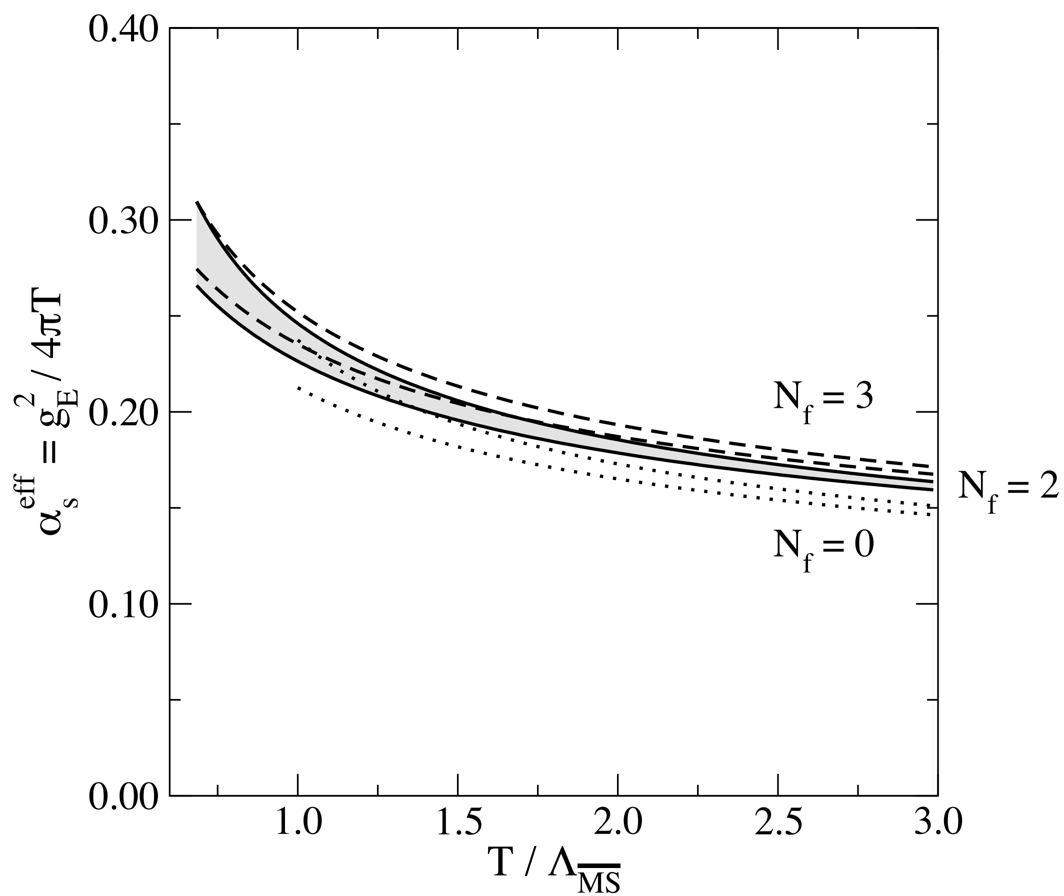
Polyakov loop ~1 in pert thy. Lattice: above 3 Td. Not Td => 3 Td.



NpQGP: electric (not magnetic), not strong coupling

Giovannangeli; Laine & Schroder: in dim.'y reduced 3D theory, compare (spatial) string tension to that in full theory. Works very well!





Only electric sector non.-pert., not magnetic.

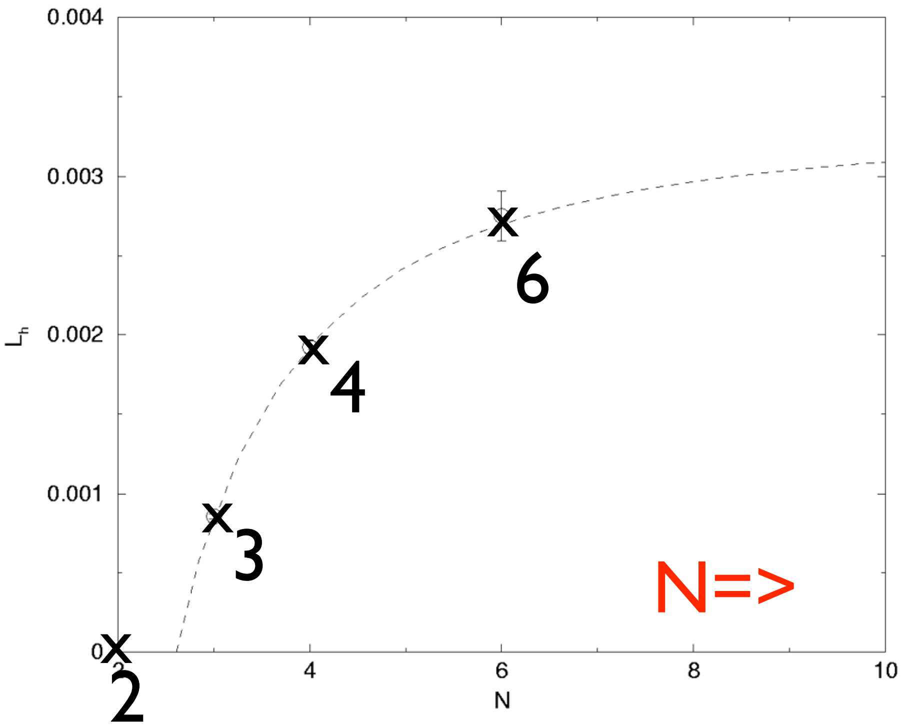
QCD: α_s at 175 MeV ~ 0.28: at T=0, mom. scale ~ 2.2 GeV.

Not (very) strong coupling!

Deconfinement: 1st order transition for N ≥ 3

Lucini, Teper, Wenger '03, '04, '05: Latent heat ~ N^2 for N= 4, 6, 8

Latent heat/
N^2



N=2: *second* order

N=3: *weakly* 1st order

N ≥ 4: *strongly* 1st order

Ordinary 1st order trans.: latent heat, masses nonzero at transition.

Perhaps: Large N near "Gross-Witten point":
transition first order, but masses vanish.

Deconfinement and the Gross-Witten point

A. Dumitru, Y. Hatta, J. Lenaghan, K. Orginos, & RDP, hep-th/0311223: DHLOP
Aharony, Marsano, Minwalla, Papadodimas, & Van Raamsdonk:
hep-th/0310285: AMMPR '03; hep-th/0502149: AMMPR '05
A. Dumitru, J. Lenaghan, & RDP, hep-ph/0410294: DLP '04
A. Dumitru, RDP, D. Zschiesche, hep-ph/0505256: DPZ '05
M.Oswald & RDP, hep-ph/0510?: OP '05

Take "pure" SU(N) gauge theory, *no* dynamical quarks.
Rigorously, a deconfining phase transition at a temperature T.

Example: scalar field invariant under a global U(1) symmetry: $\phi \to e^{i\theta}\,\phi$

Look for spontaneous breaking of U(1) symmetry through $\langle \phi \rangle \neq 0$

Start with the most general potential invariant under U(1),
use mean field theory to study phase diagram.

Mean field phase diagram

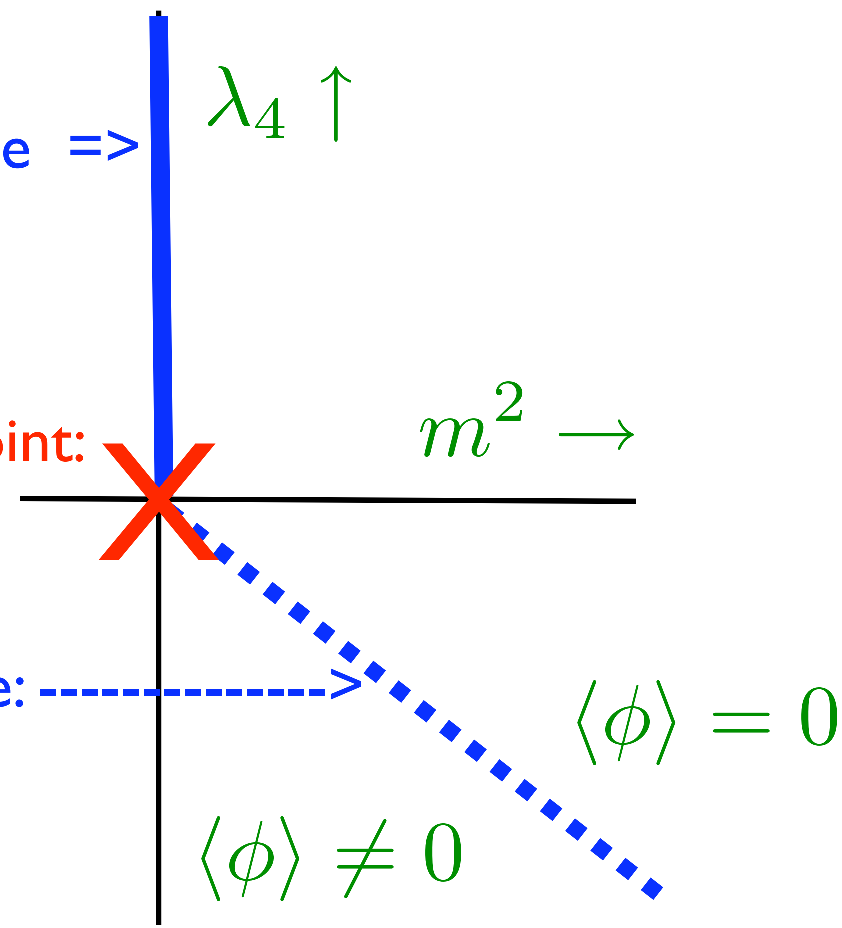
When N≠3, all phase diagrams look alike:
Lines of 1st and 2nd order transitions meet at a tri-critical point

$$\mathcal{V}_{U(1)} = m^2|\phi|^2 + \lambda_4(|\phi|^2)^2 + \lambda_6(|\phi|^2)^3 + \ldots$$

$m^2 = 0\ ,\ \lambda_4 > 0$ 2nd order line =>

$m^2 = \lambda_4 = 0$ Tri-critical point:

$m^2 > 0\ ,\ \lambda_4 < 0$ 1st order line: ------->



Matrix mean field theory

Wilson loop: $$\mathbf{L} = \mathcal{P} e^{ig \oint A_\mu dx^\mu}$$

SU(N) matrix: $$\mathbf{L}^\dagger \mathbf{L} = \mathbf{1}\ ,\ \det \mathbf{L} = 1$$

Assume invariance under local SU(N) transf.'s, Ω: $$\mathbf{L} \to \Omega^\dagger\, \mathbf{L}\, \Omega$$

Also global Z(N) symmetry: $$\mathbf{L} \to e^{2\pi i/N}\ \mathbf{L}$$

Consider transitions where Z(N) breaks, SU(N) doesn't.

Deconfinement

Start with loop in fundamental representation:

$$\ell = \frac{1}{N} \, \mathrm{tr} \, \mathbf{L}$$

T ≠ 0: thermal Wilson line => Polyakov loop. Invariant under SU(N).

Fundamental loop carries Z(N) charge; ~ (trace) "test" quark propagator.

Z(N) symmetric = confined: $\langle \ell \rangle = 0 \; , \; T < T_d$

Z(N) sym. broken = deconfined: $\langle \ell \rangle \neq 0 \; , \; T > T_d$

Deconfining transition at T_d

Matrix models

Matrix in the measure:

$$\mathcal{Z} = \int d\mathbf{L}\ \exp(-\mathcal{V})$$

Adjoint loop:

$$\ell_{adj} = \frac{1}{N^2-1}\left(|\mathrm{tr}\mathbf{L}|^2 - 1\right)$$

Z(N) charge: fundamental loop = charge 1. Adjoint loop = charge 0.

Most general potential sum of Z(N) *neutral* loops:

$$\mathcal{V} = m^2\ \ell_{adj} + \Sigma_j\ \kappa_j \ell_j\ ,\ e_j = 0$$

Adjoint loop "mass" term. Higher loops "interactions"

Large N matrix models

At large N, "factorization" =>

$$\ell_{adj} \approx |\ell|^2 + 1/N^2$$

Assume loop potential powers of the fundamental loop:

$$\mathcal{V}/N^2 = m^2|\ell|^2 + \kappa_4(|\ell|^2)^2 + \kappa_6(|\ell|^2)^3 + \ldots$$

At large N:

confined phase: $\langle\ell\rangle = 0 \; , \; \langle\mathcal{V}\rangle/N^2 = 0$

deconfined phase: $\langle\ell\rangle \neq 0 \; , \; \langle\mathcal{V}\rangle/N^2 \sim 1$

$\langle\mathcal{V}\rangle$ ~ free energy: ~ N^2 from deconfined gluons, ~1 from hadrons.

Large N: Vandermonde "potential"

Brezin, Ityzkson, Parisi & Zuber '78; Gross & Witten '81
Kogut, Snow & Stone = KSS '82; Green & Karsch '84
AAMPR '03, '05. DHLOP '03. DLP '04.

Choose $\ell = \mathrm{tr}\,\mathbf{L}/N$ real & positive; minimize with respect to eigenvalues of $\mathbf{L}$

Measure of matrix integral includes Vandermonde determinant
=> Vandermonde "potential":

$$\mathcal{V}_{Vdm}/N^2 = +\,\ell^2 \quad , \quad \ell < \frac{1}{2}$$

$$\mathcal{V}_{Vdm}/N^2 = -\frac{1}{2}\log\left(2\left(1-\ell\right)\right) + \frac{1}{4} \quad , \quad \ell > \frac{1}{2}$$

GW: the Vdm potential is discontinuous, of *third* order, at $\ell = 1/2$

Gross-Witten point

Potentials ~ N^2 => at infinite N, vacua minima of

$$\mathcal{V}_{eff} = \mathcal{V} + \mathcal{V}_{Vdm}$$

Introduce $\widetilde{m}^2 = m^2 + 1$

For $\ell < 1/2$

$$\mathcal{V}_{eff}/N^2 = +\,\widetilde{m}^2\ \ell^2$$

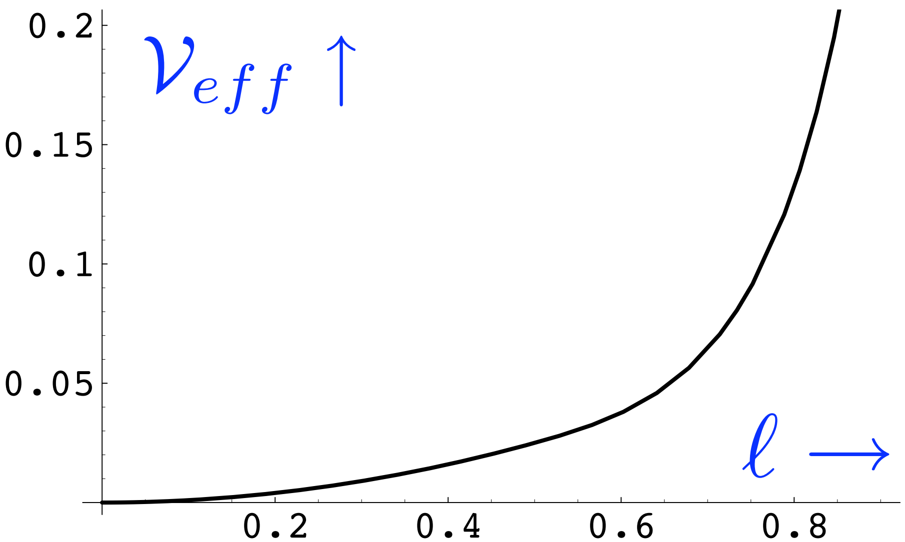
$\widetilde{m}^2$ > 0: confined phase. < 0: deconfined phase.

Gross-Witten point: $\widetilde{m}^2 = 0\ ,\ \kappa_4 = \kappa_6 = \ldots = 0$

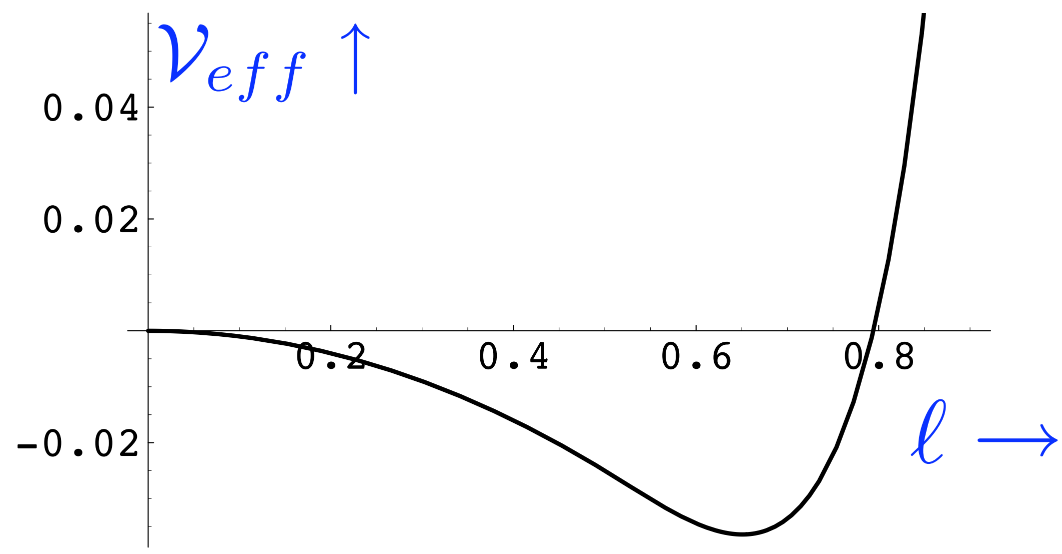
Only non-trivial because of Vandermonde potential.
GW point unnatural: infinite number of couplings tuned to vanish.

Near the GW point

All potentials have 3rd order discontinuity at $\ell = 1/2$



<= confined: $\widetilde{m}^2 = +.1$



deconfined=>

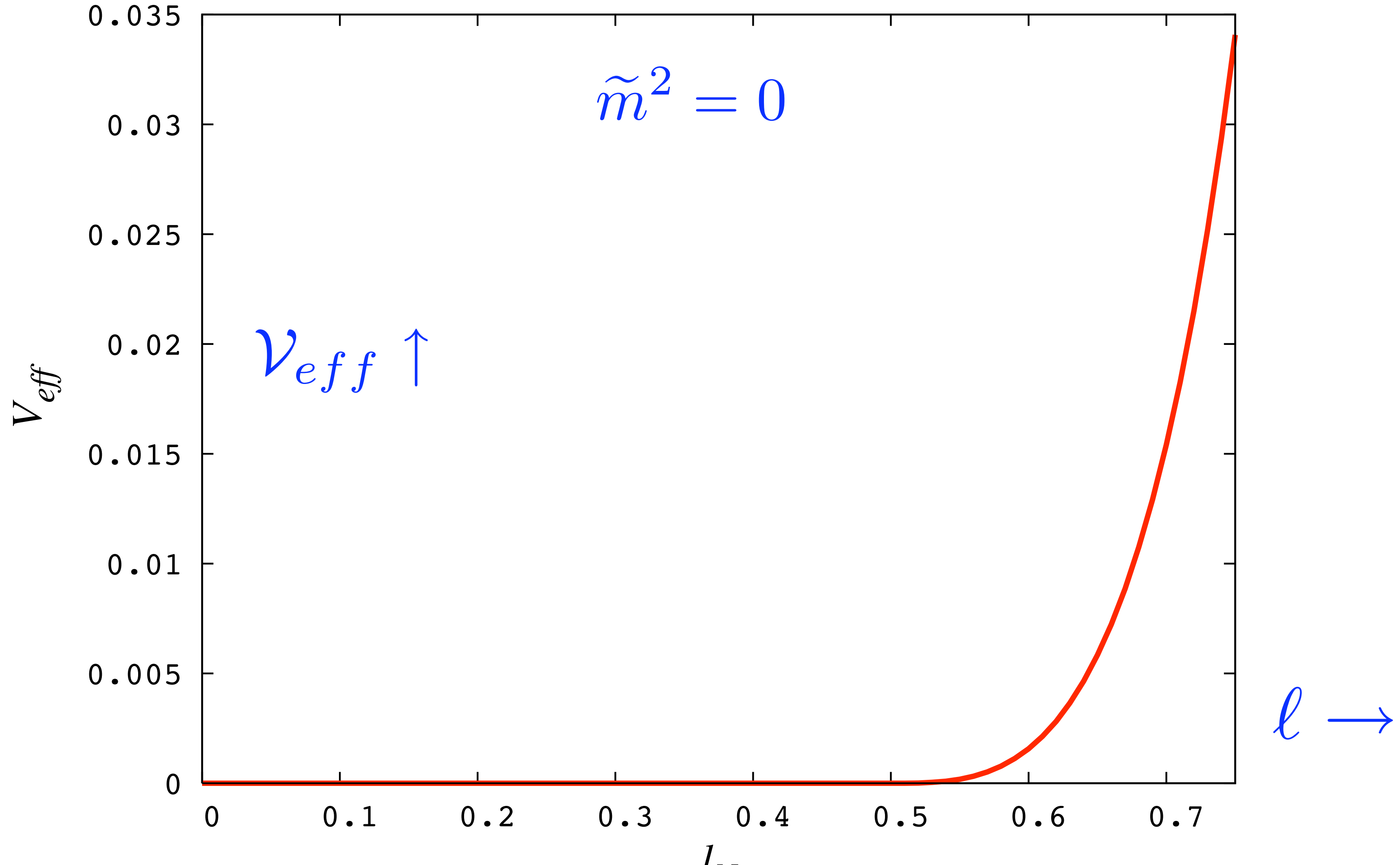
$\widetilde{m}^2 = -.1$

At the GW point

At transition: order parameter jumps: $\langle \ell \rangle \; : \; 0 \rightarrow 1/2$ Latent heat nonzero

And masses vanish (asymmetrically) => "critical" 1st order transition

New minimum = 3rd order discontinuity at 1/2

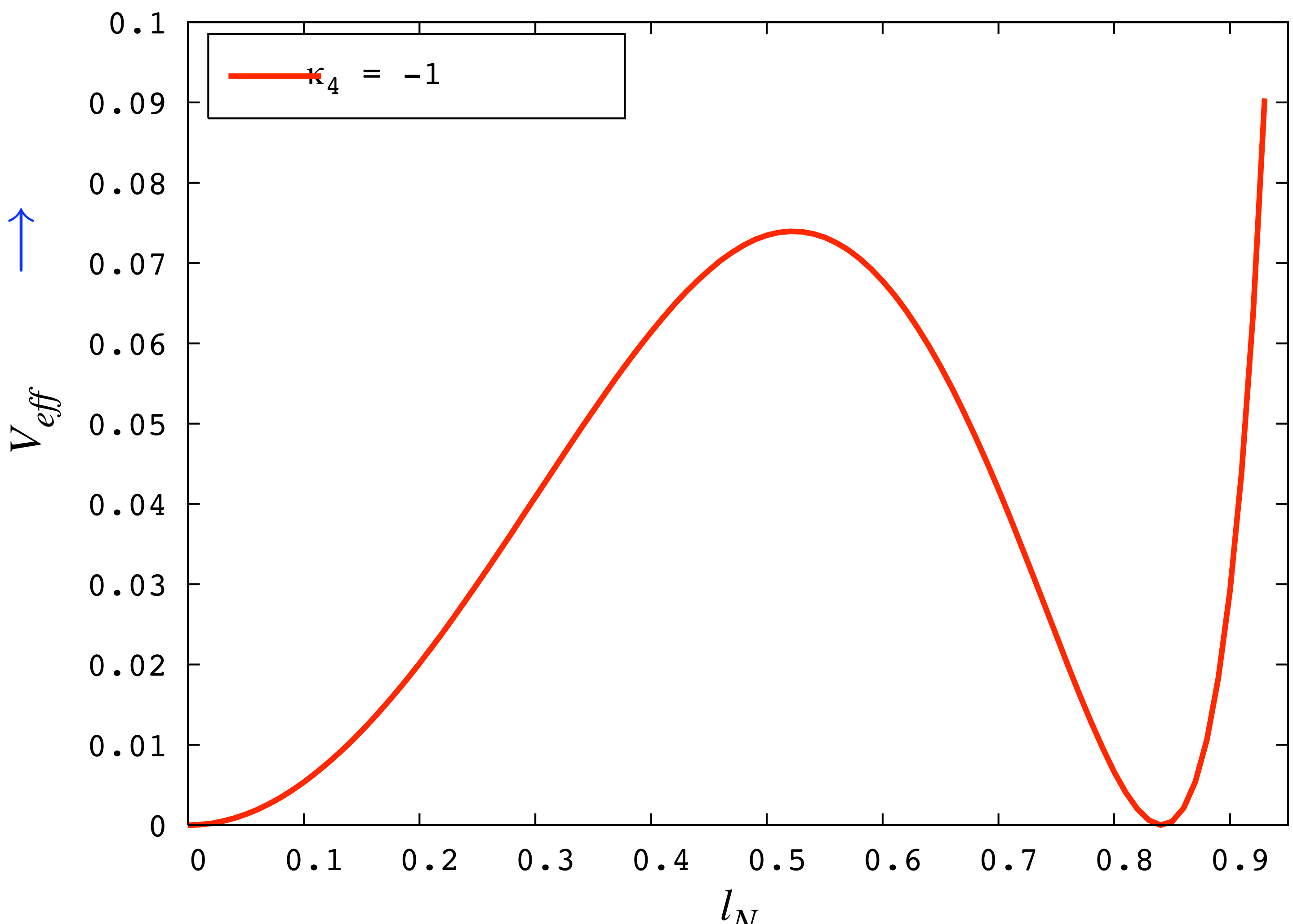


Away from the GW point

Add negative quartic coupling: $\mathcal{V}/N^2 = m^2|\ell|^2 - (|\ell|^2)^2$

Typical strongly 1st order transition: masses nonzero at transition (below)
New minimum ≠ 3rd order discontinuity at 1/2

$\mathcal{V}_{eff} \uparrow$



$\ell \longrightarrow$

GW = "ultra"-critical point

Phase diagram: tri-critical => Gross-Witten point.

$$\mathcal{V}_{eff}/N^2 = \widetilde{m}^2|\ell|^2 + \kappa_4(|\ell|^2)^2 + \kappa_6(|\ell|^2)^3 + \ldots \qquad \ell < 1/2$$

Away from GW point,

ordinary 1st or 2nd order transitions.

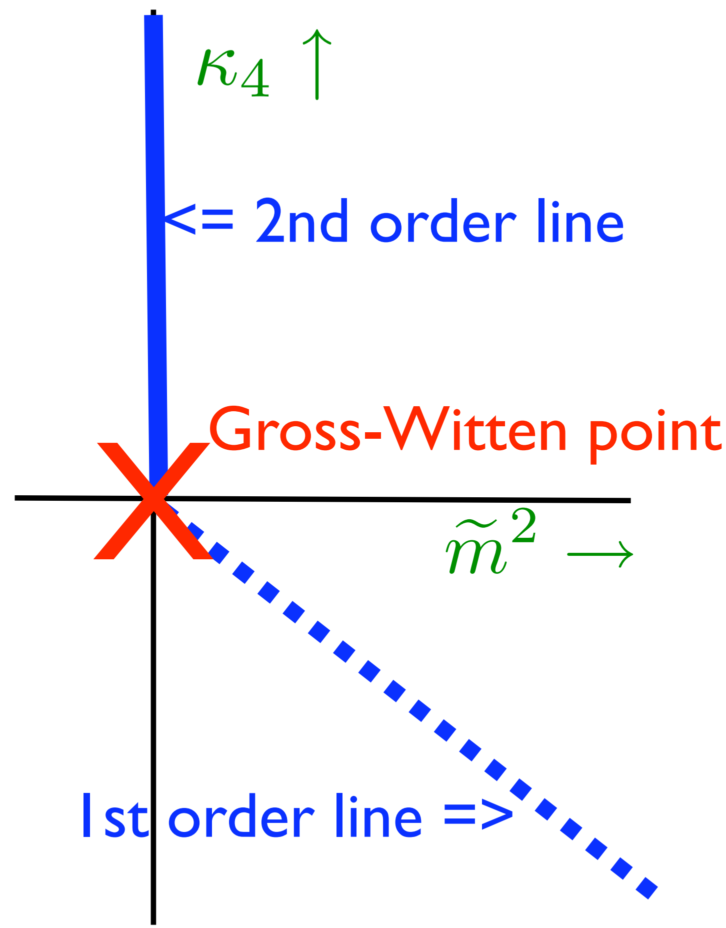
Only at GW point:

Nonzero latent heat, jump in order parameter

AND zero masses

"Ultra"-critical as infinite # couplings vanish

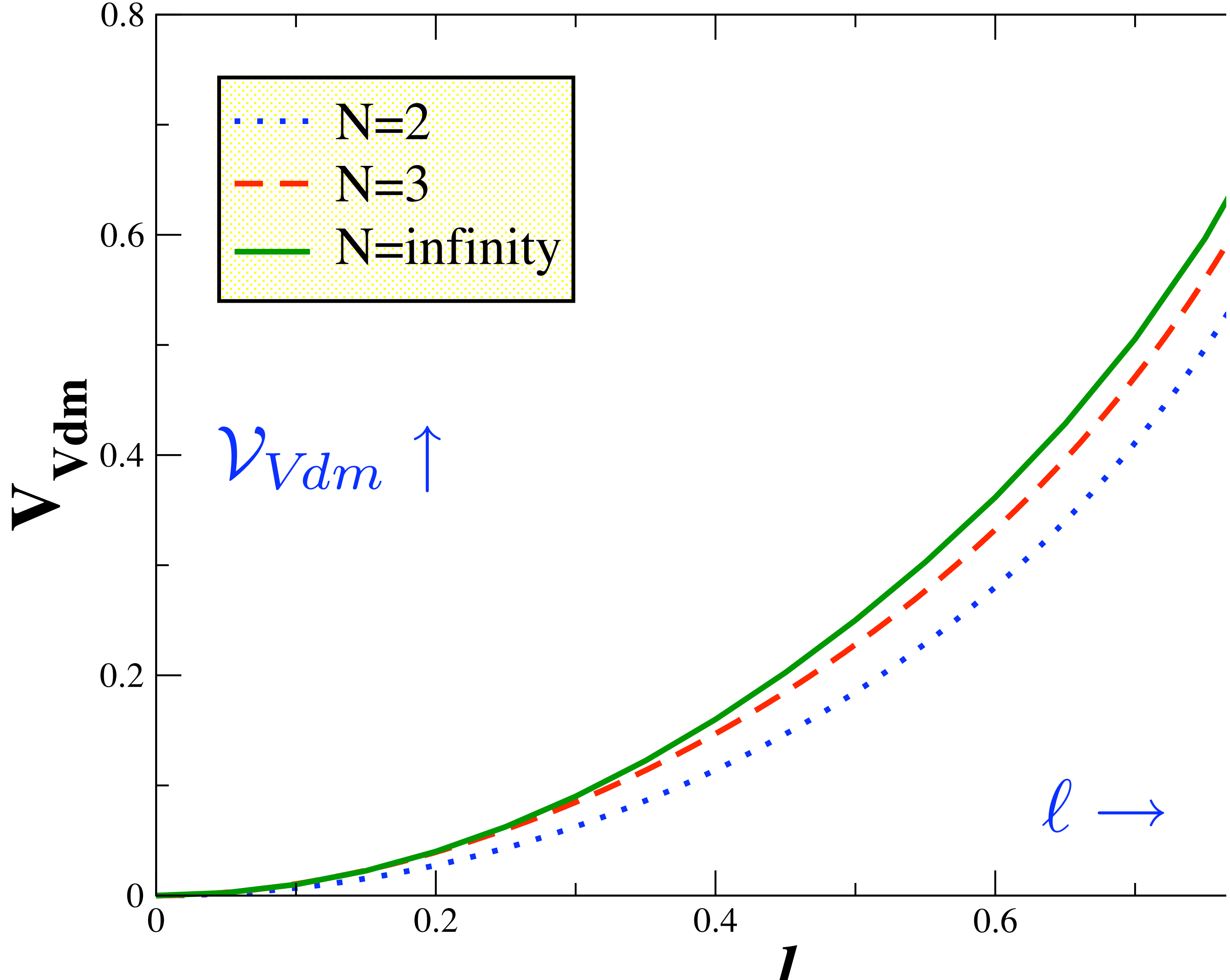
AMMPR '03, DLP '04



Finite N: Vandermonde potential

Infinite N: discontinuity of 3rd order at 1/2. Continuous at finite N.

Numerically, N=2 and 3 *close* to infinite N. DLP '04



N = 3: matrix models

Finite N: Gross-Witten pt = *ordinary* 1st order transition, masses always ≠ 0

N=3: triplet loop with Z(3) charge

Z(3) neutral loops: octet, decuplet... Write potential as:

$$\mathcal{V}/8 = m^2|\ell_3|^2 + \kappa_3((\ell_3)^3 + \mathrm{c.c.}) + \ldots$$

Cubic invariant => transition always 1st order Svetitsky & Yaffe '82

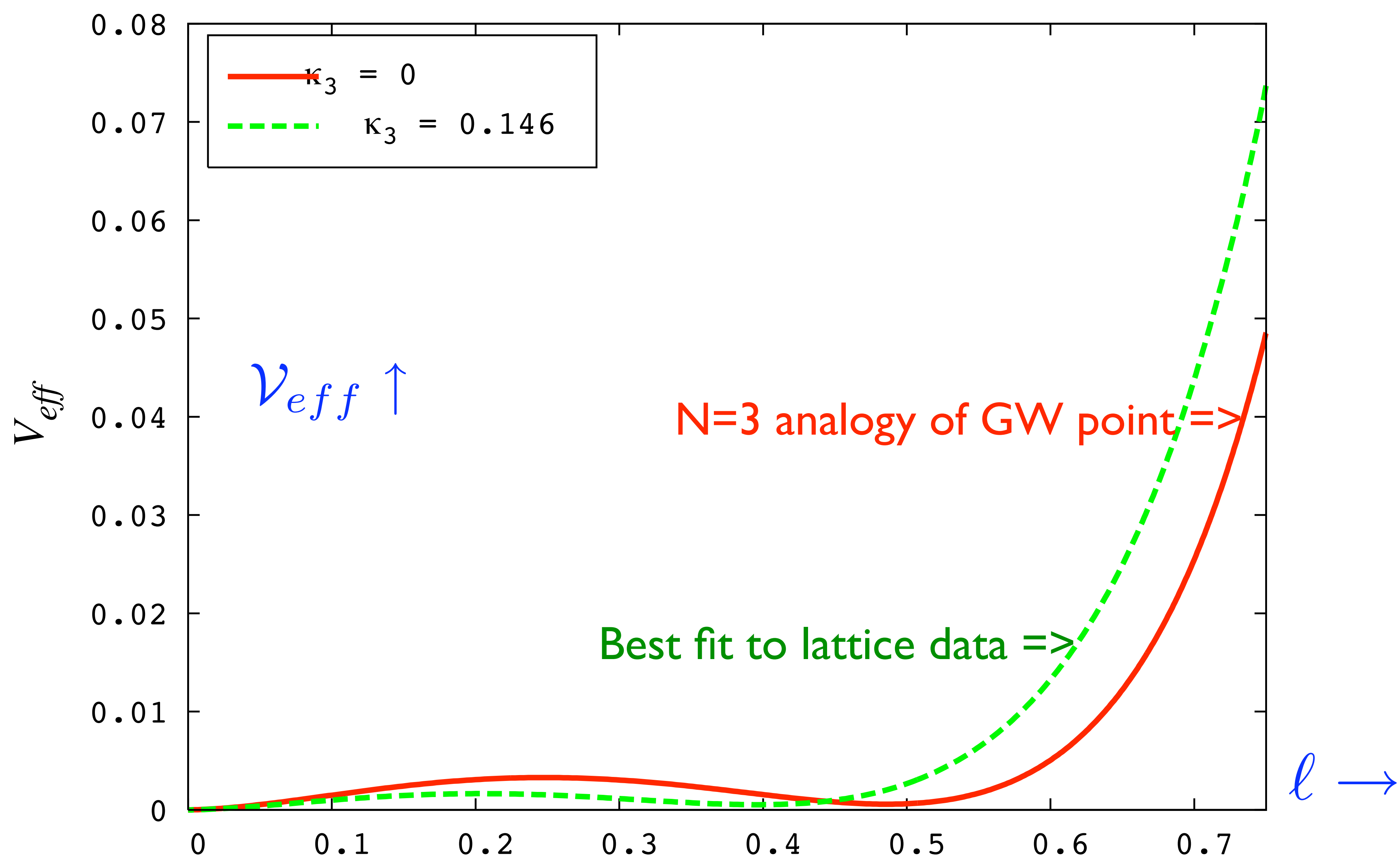
KSS '82: at N=3 analogy of GW pt, jump in $\langle \ell \rangle$ to .485 ~ 1/2

DLP '04: fit to lattice data for renormalized triplet loop (shown later)

Lattice: $\langle \ell \rangle$ jumps to ~ .4 at T_d => N=3 transition *close* to N=3 GW point.

Lattice: N = 3 close to GW point

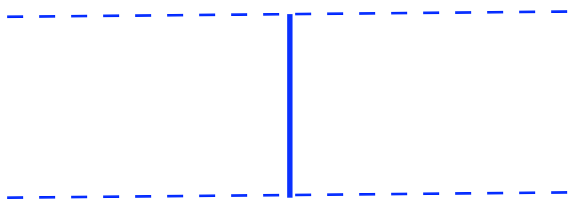
Take ren'd loops from lattice data.
Fit matrix model, with $m^2 \sim T_d - T$ Only need *small* cubic term. DLP '04



Renormalized Polyakov Loops

Gervais & Neveu '80. Polyakov '80. Dotsenko & Vergeles '80....
Kaczmarek, Karsch, Petreczsky & Zantow = KKPZ '02 +... DHLOP '03.

Loop with no cusps:



Loop with four cusps:

τ↑: imaginary time,
0 => 1/T

Four dim.'s: loops of length L renormalize by "mass" ren. (R = irreducible rep.)

$$\widetilde{\ell}_R = \mathcal{Z}_R\, \ell_R \quad , \quad \mathcal{Z}_R = \exp(-m_R^{div} L + \gamma_{cusp})$$

Divergent mass:

"a"=lattice spacing, C_R = Casimir: $a\, m_R^{div} = +C_R\, g^2 (1 + \# g^2 + \ldots)$

Anomalous dimension γ=0 for straight loops; ≠ 0 with cusps.

Ren.'d Polyakov loops on lattice

DHLOP '03: compare two lattices, *same* temperature, *different* lattice spacing.

N_t = 1/(aT) changes => obtain $a\, m_R^{div}$, ren'd loop:

$$- \log\left(|\langle \ell_R \rangle|\right) = a\, m_R^{div}\; N_t + f_R^{cont} + f_R^{lat}/N_t + \ldots$$

$$|\langle \widetilde{\ell}_R \rangle| = e^{-f_R^{cont}} + \ldots$$

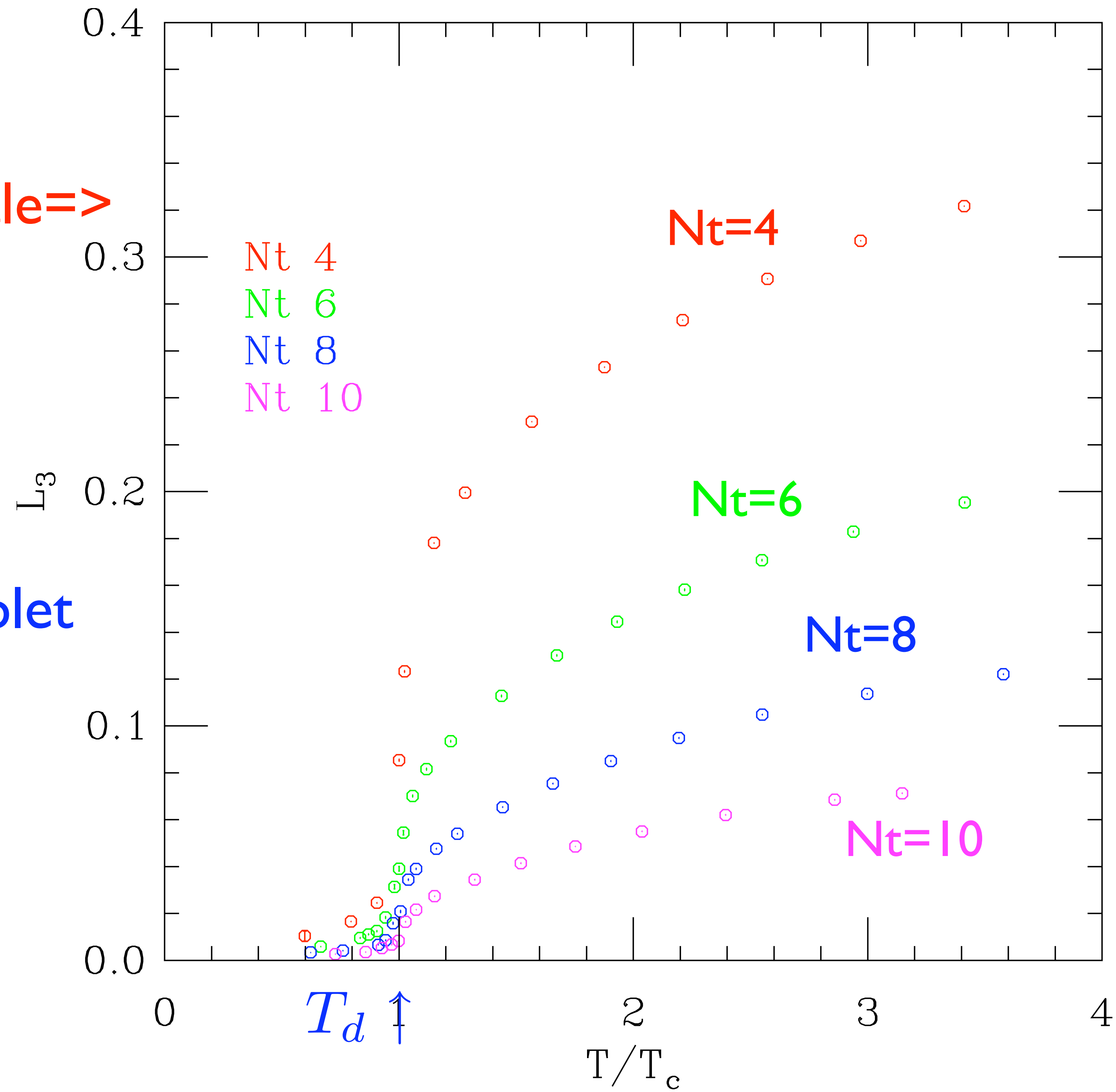
Find $f_R^{lat} \approx 0$

Coupling for transition changes with N_t
=> to obtain the same T at different N_t, must compute at different β.
Doable, not trivial.

SU(3) Wilson action, N_t = 4,6,8,10; # spatial steps = 3 N_t
Lattice coupling constant β = 6/g^2: related to temperature by Non-Pert. Ren.

Bare triplet loop vs T, Nt

Note scale=>
~ .3



Bare triplet loop↑

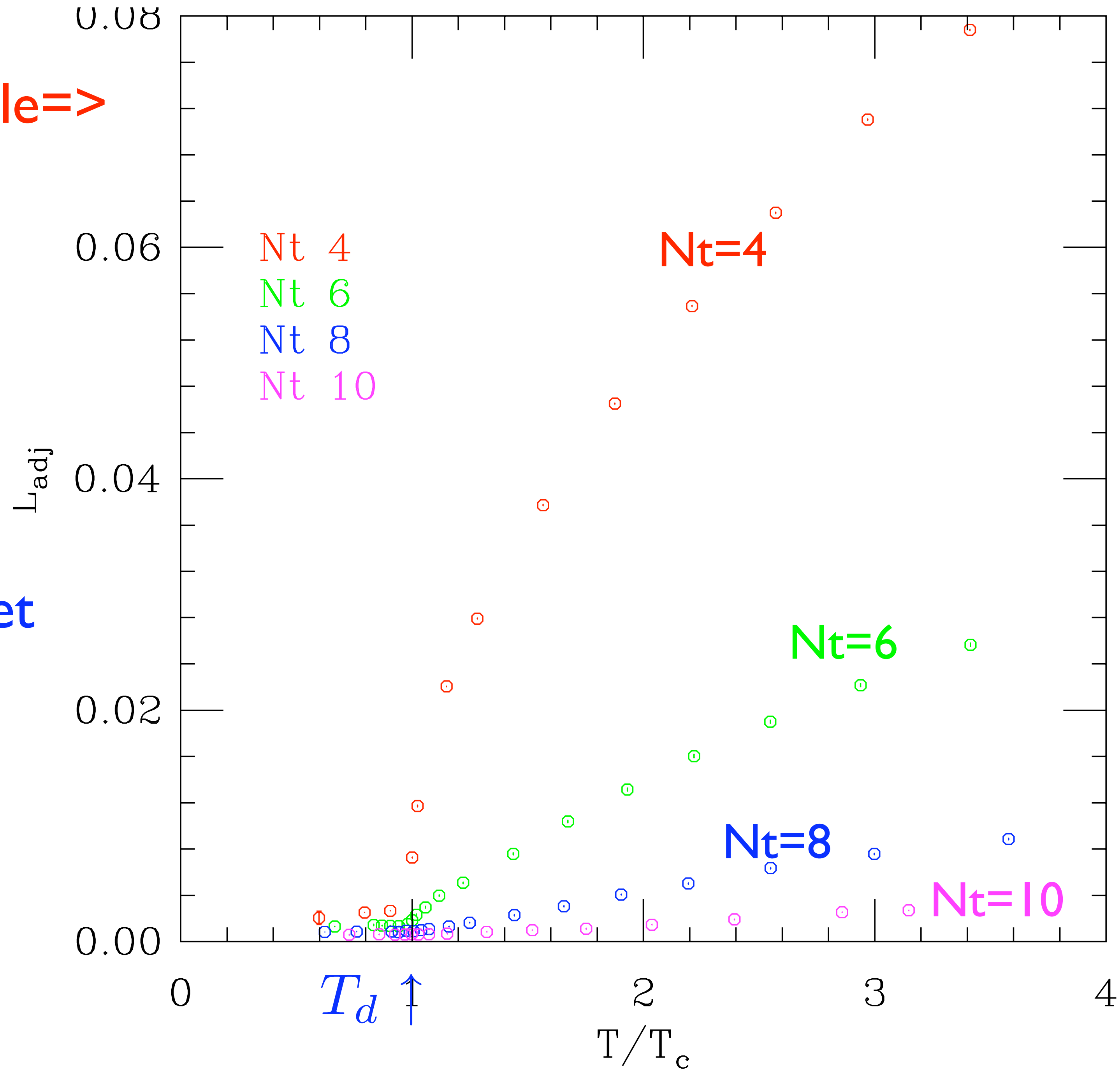
Nt = # time steps.

Bare loop vanishes as Nt → ∞

$T/T_d \rightarrow$

Bare octet loop vs T, Nt

Note scale=>
~ .06



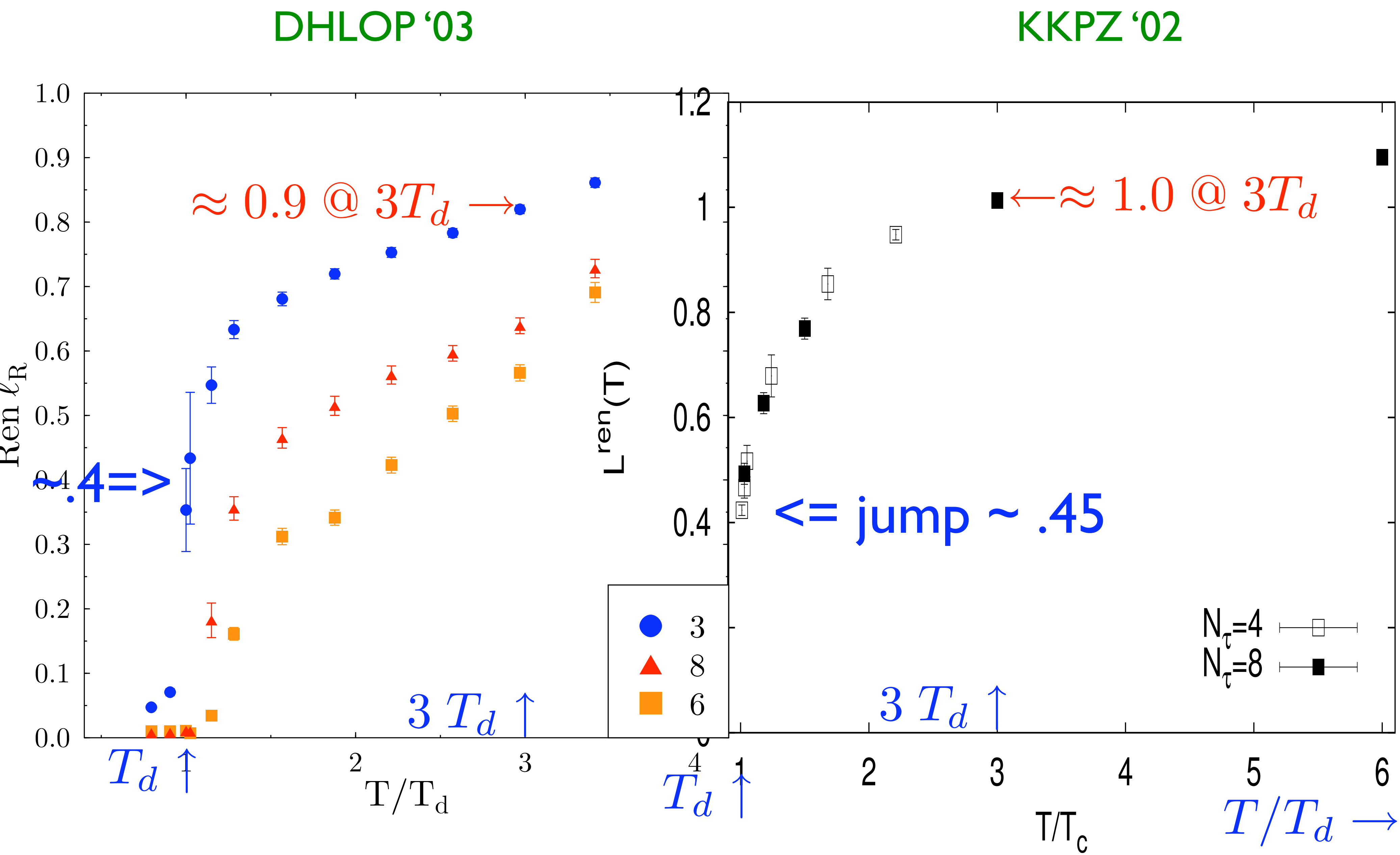
Bare octet
loop↑

Sextet loop
very similar

Decuplet
loop only
measurable
at Nt=4

$T/T_d \rightarrow$

Lattice SU(3): Renormalized Polyakov loops



Agree to ~ 10%: difference due to cusp renormalization?

Lattice: SU(3) ≈ SU(∞) to ~25%

At large N, “factorization” => all loops product of fundamental (& anti-fund.)

Migdal & Makeenko ‘80, Eguchi & Kawai ‘82...Gross & Taylor ‘93

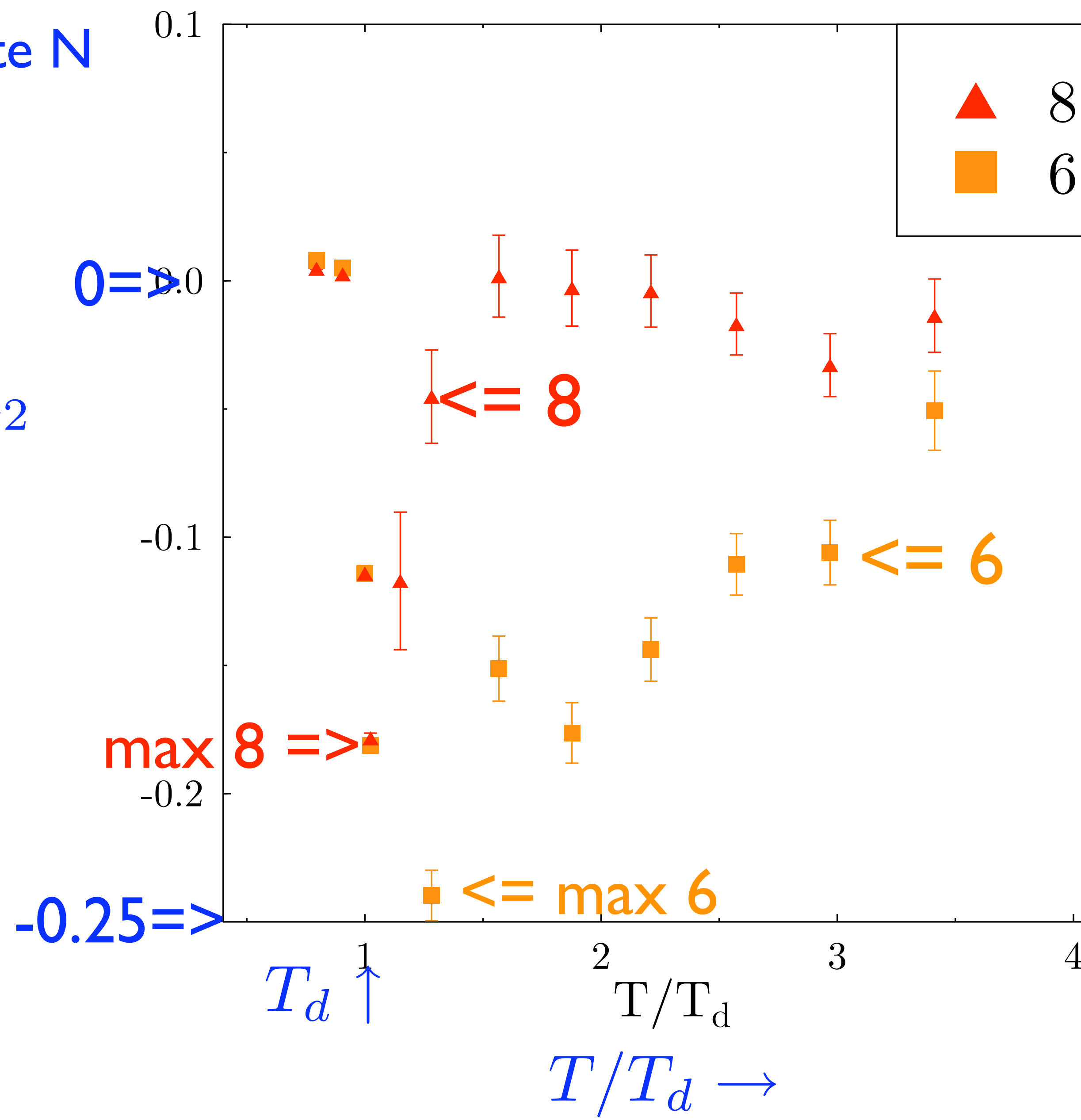
“Difference” loops vanish at infinite N

$$\delta\ell_6 \equiv \langle \ell_6 \rangle - \langle \ell_3 \rangle^2 \sim 1/N$$

$$\delta\ell_8 \equiv \langle \ell_8 \rangle - |\langle \ell_3 \rangle|^2 \sim 1/N^2$$

Lattice:
Corrections to factorization
very small, except near T_d

Above T_d:
“spikes” in difference loops



Lattice: String tension vs. T, N= 3, 4 & 6

Confined phase: string tension at T≠0/at T=0 (y-axis)

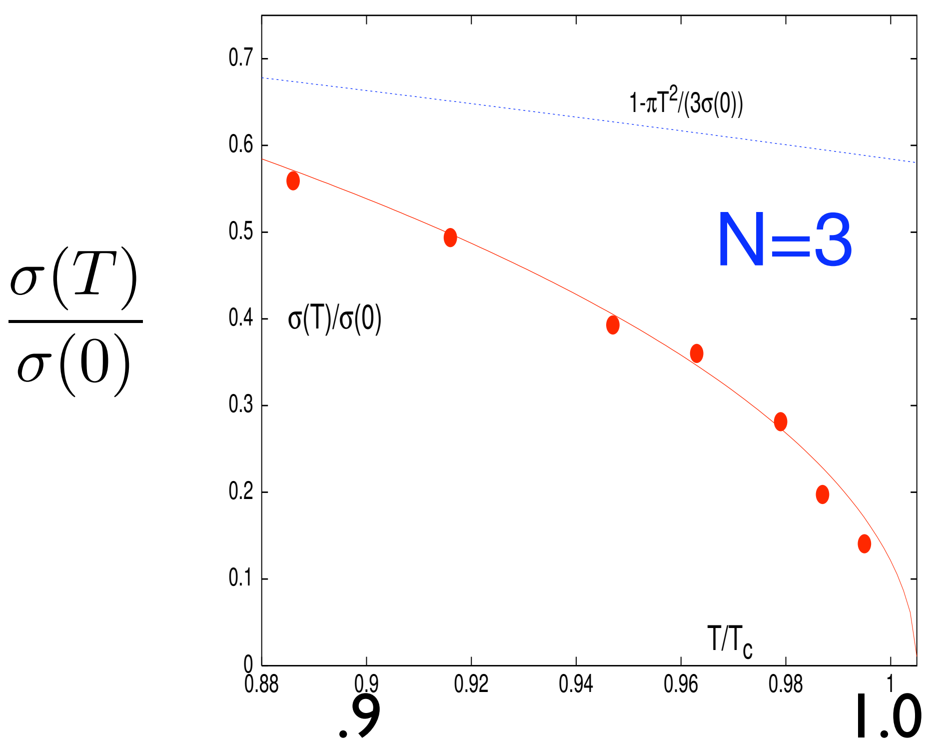
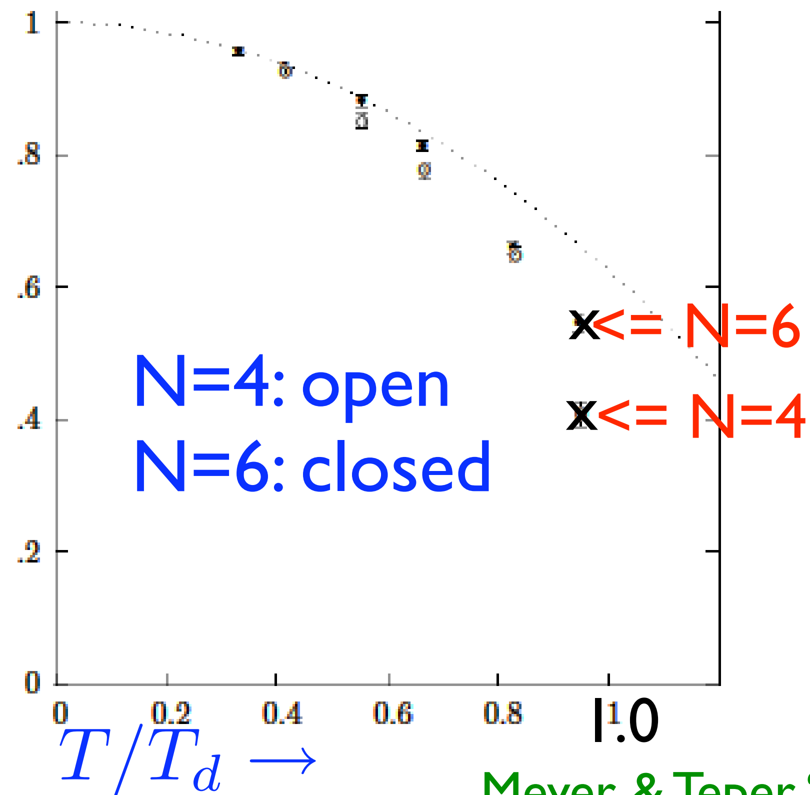


FIG. 1:



Meyer & Teper ‘04

At fixed T/Td, ratio increases with N.

Lattice: for N= 2,3,4, $\sigma(T_{1/2}) \equiv 0.5\,\sigma(0) \;:\; (T_d - T_{1/2})/T_d \sim 0.8/N^2$

Window, ~ 1/N^2, where GW point is infrared stable fixed point?

Nonzero quark density

Quarks act like background Z(3) field, ~ real part of loop.

Quark chemical potential, μ: background field for imaginary part of loop, with *imaginary* coefficient! Karsch & Wyld '86, DPZ '05

$$V_{qk} = -\frac{h}{2}(e^{\mu}\ \ell_3 + e^{-\mu}\ \ell_{\overline{3}}) = -h(\cosh(\mu)\ \mathrm{Re}\,\ell_3 + i\sinh(\mu)\ \mathrm{Im}\,\ell_3)$$

In matrix model: sum over both L and charge conjugate, L^*.

After summation, *all* contributions to partition function explicitly real.

Although both v.e.v's real, unequal: $\langle \ell_3 \rangle \neq \langle \ell_3^* \rangle$

Generalizes to dynamical quarks on lattice: sum over charge conjugate lattice.

Matrix model: about μ=0, one v.e.v. increases, the other decreases.
Test of lattice methods.

Fluctuations in matrix model

Infinity of "kinetic" terms. Three simplest couplings:

$$\mathcal{L} = \frac{1}{g^2}\,\mathrm{tr}\,|\partial_i \mathbf{L}|^2 \left(1 + \frac{3\xi}{2g^2}(1-\ell_{ad})\right) + \frac{4\lambda}{g^4}\,|\partial_i \ell_N|^2$$

Looks like generalized non-linear sigma model:

$$\mathbf{L}^\dagger \mathbf{L} = \mathbf{1} \ \ , \ \ \det \mathbf{L} = 1 \ \ , \ \ \ell_N = \mathrm{tr}\,\mathbf{L}/N \ \ , \ \ \ell_{ad} = (1 - |\mathrm{tr}\mathbf{L}|^2)/(N^2-1)$$

Compute β-functions in two spacetime dimensions: OP '05

$$\beta(g^2) \sim -g^4 \ \ , \ \ \beta(\xi) \sim -g^2\lambda \ \ , \ \ \beta(\lambda) \sim +g^2\lambda$$

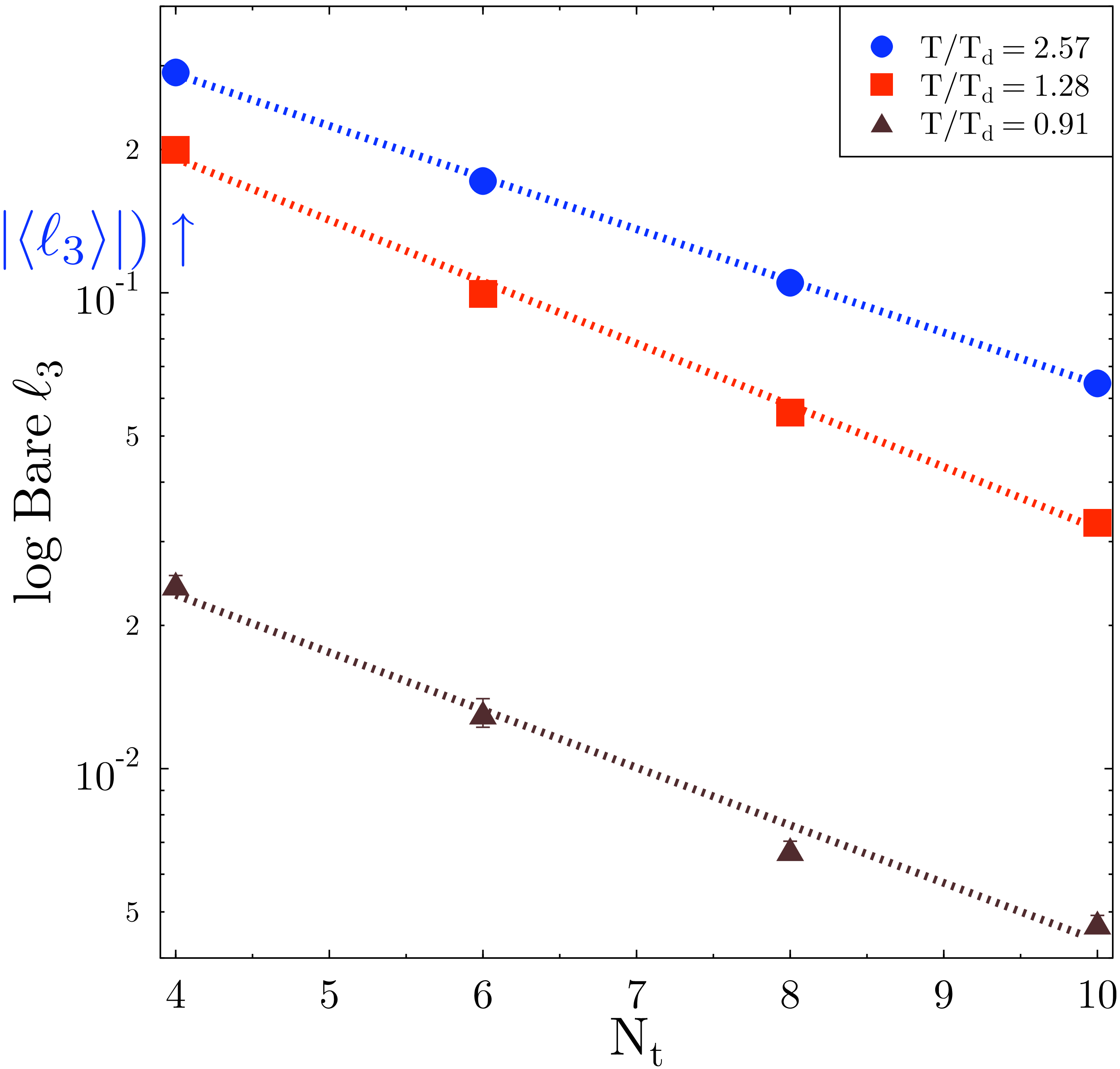
Two out of three couplings asymptotically free.

Shows eff. theory of Wilson lines, for 2+1 dimensions, sensible in pert. thy.

"A possible eureka."

Mass div.'s exponentiate: log(bare loop) vs. N_t

$\log(|\langle \ell_3 \rangle|) \uparrow$



$N_t \longrightarrow$

<= Test of exponentiation of mass divergences.

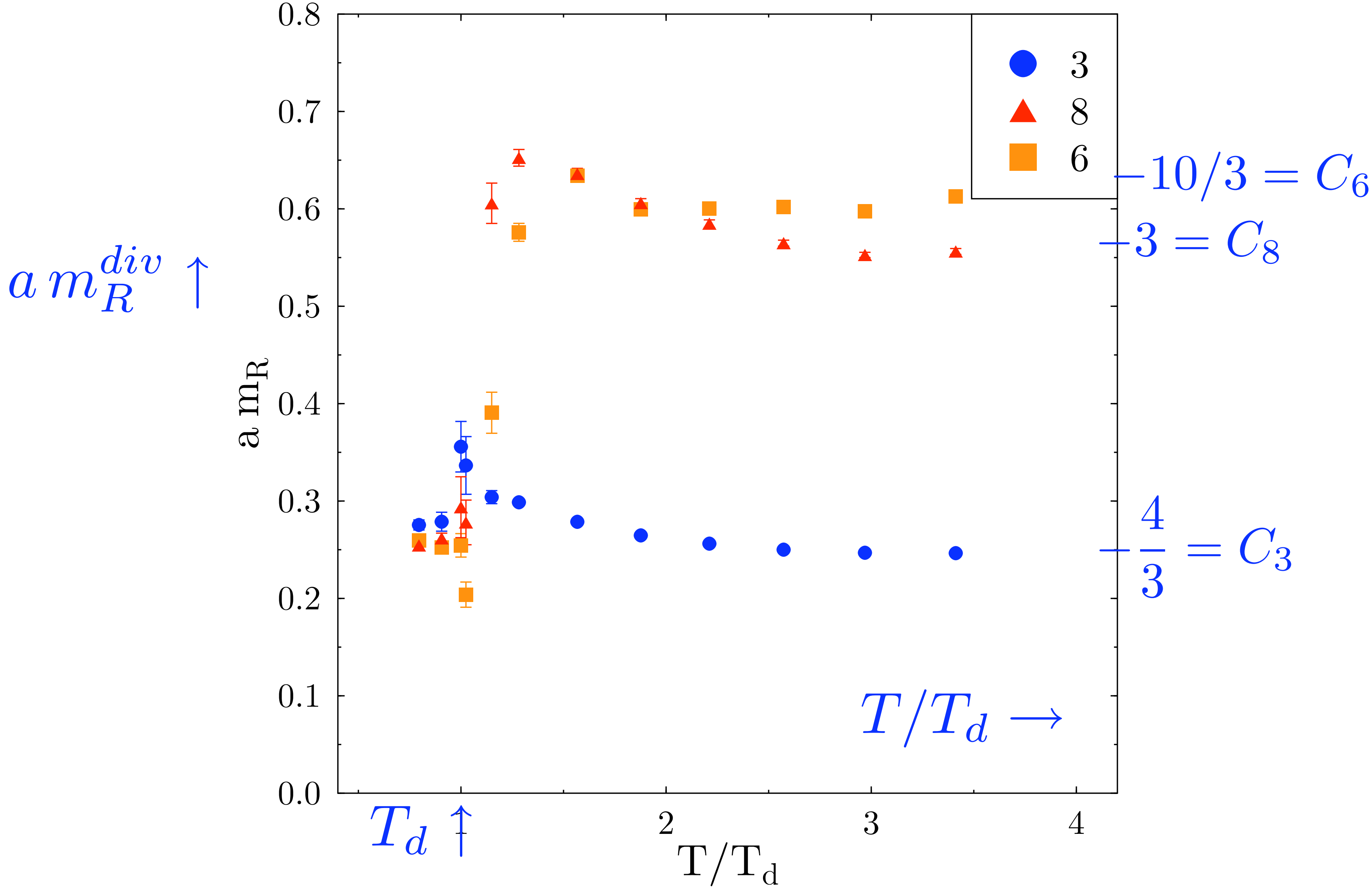
No evidence seen for 1/N_t corrections; must be there.

Lattice SU(3): divergent masses

DHLOP: Triplet, sextet, octet loops.

KKPZ: Triplet loop, Z_R from short distance behavior of two-point functions.

Casimir scaling of divergent masses at 3 Td.



Bare loops don't factorize

Bare octet difference loop/bare octet loop: violations of factor. 50% @ Nt =4 200% @ Nt = 10.